Study on the Optimal Design of Soccer Robot based on the Mechanical Analysis

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Abstract: Estimation of matrix eigenvalues is one of the important problems in matrix theory. In this paper, the upper bound of the maximum eigenvalue of Hadamard product of a nonnegative matrix is estimated by using the method of estimating the eigenvalue of a matrix in an elliptic region. By using the maximum value of the sum of rows of matrix elements except principal diagonal elements, the lower bounds of the minimum eigenvalues of the Hadamard product of a matrix, an inverse matrix and two matrices are estimated. In this paper, new estimators of the minimum eigenvalue lower bound of the inverse matrix of a nonsingular matrix and the Hadamard product of a matrix are given. These estimators only depend on the elements of the matrix. Due to the deep research of non-negative matrices, the study of non-negative matrix Hadamard products has also been broadened. The estimation of the upper bound of the maximum eigenvalue of the matrix Hadamard product has been successively obtained many results, which depend on the maximum eigenvalue and the main diagonal element of the two non-negative matrices. The matrix Hadamard product determinant inequality has obtained the results of some semi-positive definite matrix Hadamard product determinant inequalities and the results of some matrix Hadamard product determinant inequalities.

1. Introduction

The Hadmard product of matrices is the product of special matrices. The estimation of the minimum eigenvalue lower bound of the Hadmard product of matrices and inverse matrices has been extensively studied [1]. Matrix is not only one of the important research topics of Computational Mathematics and matrix theory, but also has important practical value in many fields such as biology, physics, Economic Mathematics and so on. The problem of estimating the minimum eigenvalue lower bound of Hadmard product of matrix and its inverse matrix has been extensively concerned and studied, and a series of estimators [2] have been obtained. The Hadamard product of matrices is a special matrix product, which refers to an algorithm for obtaining new matrices by multiplying the corresponding elements of two matrices with the same row number and column number. The matrix Hadamard product is a special matrix product, which is widely used in the study of the weak and small principles of eigenfunctions and partial differential equations in probability theory [3]. Due to the deep research of non-negative matrices, the study of non-negative matrix Hadamard products has also been broadened. It is widely used in the product of the integral equation kernel, the study of the eigenfunction in probability theory, the weak minimal principle of partial differential equations, and the combination scheme in combination theory.

The matrix is a matrix closely related to the non-negative matrix, and the study of the Hadamard product of the matrix has been developed accordingly. Since the M-matrix is a special matrix with important application background, the estimation of the lower bound of the minimum eigenvalue of the Hadamard product of the M-matrix and its inverse matrix has become one of the concerns [4]. In this paper, some new estimators for the lower bounds of the minimum eigenvalues of the Hadamard product of the M-matrix and its inverse matrix are given. Theoretical analysis and numerical examples show that the new estimator improves some existing results [5]. Since the 1980s, many results have been obtained for estimating the upper bound of the maximum eigenvalue of matrix Hadamard product. These results depend on the maximum eigenvalue of two nonnegative matrices and the principal diagonal element [6]. Matrix is a kind of real matrix with positive principal diagonal elements and non-principal diagonal elements. It has important applications in biology, physics, economics and dynamic systems [7]. Matrix Hadamard product determinant inequalities have yielded results of some semi-definite matrix Hadamard product determinant inequalities and some matrix Hadamard product determinant inequalities.

2. Estimation of eigenvalues of Hadamard product of Nonnegative Matrices

Matrix is an important matrix in computational mathematics, and it has a wide range of applications. It has extensive links to biology, physics, mathematics, and social sciences in the natural sciences. $R^{m\times n}$ is used to represent the m×n-order real matrix set, $C^{m\times n}$ is the m×n-order complex matrix set, and $\rho(P)$ is the Perron eigenvalue of the n×n-order non-negative matrix P. Let $A=(a_{ij})\in R^{m\times n}$, if A can be expressed as $A=\lambda I-B$, where all elements in B are not less than 0, and when $\lambda\geq\rho(B)$, then A is called matrix. In particular, when $\lambda>\rho(B)$, A is called a non-singular matrix. When $\lambda = \rho$ (B), A is called a singular matrix. Characteristic polynomial coefficients of matrices play a central role in quantum physics applications, especially as part of the partition function needed to study the structure of neutron stars and the thermodynamic properties of Fermi subsystems generated during their evolution [8]. If the result of the eigenvalue problem is sensitive to the small perturbation of the initial data, it is called ill-conditioned, otherwise it is called good-conditioned. The data standard of this morbidity degree is described by condition number. The morbidity degree of coefficient disturbance is described by similar condition number below.

The importance of matrix Hadamard product has been recognized by more and more mathematicians. As two special types of matrices, nonnegative matrices and matrices have been extensively studied in recent years due to their wide application. In this paper, we mainly study the estimation of maximum eigenvalue and upper bound of spectral radius of Hadamard product of two nonnegative matrices and the estimation of lower bound of minimum eigenvalue of Hadamard product of matrix and inverse matrix of another matrix. Two new estimators of maximum eigenvalue and upper bound of spectral radius of Hadamard product of two non-negative matrices are given by using the method of estimating eigenvalues of matrices in elliptic region. An example shows that the estimator is more accurate than the existing estimators. Applying to the perturbation boundary problem of matrix feature polynomial coefficients on eigenvalue changes, the coefficient perturbation bound is quickly obtained [9]. The Hadamard product of a matrix is a special matrix product, which refers to an algorithm that multiplies two corresponding elements of the same number of rows and columns to obtain a new matrix. It is widely used in numerical analysis, biology, mathematical finance, operations research, etc., making mathematic researchers have a strong interest in it.

Due to the deep research of non-negative matrices, the study of non-negative matrix Hadamard products has been broadened. It is widely used in the product of integral equation kernels, the study of eigenfunctions in probability theory, the weak mini-principle of partial differential equations, Areas such as combination schemes in combination theory. The matrix is a matrix closely related to the non-negative matrix, and the study of the Hadamard product of the matrix has been developed accordingly. In recent years, there have been many studies on non-negative matrix and matrix Hadamard product, mainly focusing on the estimation of spectral radius of non-negative matrix Hadamard product. Matrix Hadamard product determinant inequalities have yielded some results of semi-definite Hermite matrix Hadamard product determinant inequalities and some results of matrix Hadamard product determinant inequalities. The estimation of the upper bound of the maximum eigenvalue of Hadamard product of nonnegative matrices has obtained many results since the 1980s. These results depend on the maximum eigenvalues and principal diagonal elements of two nonnegative matrices for column diagonally dominant nonnegative matrices.

3. Estimation of Eigenvalues of Matrix Hadamard Product

Matrix is not only one of the important research topics of Computational Mathematics and matrix theory, but also has important practical value in many fields such as biology, physics, Economic Mathematics and so on. Matrix Hadamard product is a special matrix product. It is widely used in the study of eigenfunction in probability theory and weak minimization principle in partial differential equation. Influenced by these application backgrounds, recently, many experts and scholars have extensively discussed the minimum eigenvalue bounds of Hadamard product of nonsingular matrices, and given some good estimates. Matrix is a kind of positive matrix with all the main diagonal elements and non-positive diagonal elements. It has important applications in the fields of biology, physics, economics and dynamic systems. In recent years, the estimation of the lower bound of the minimum eigenvalue of the Hadamard product of the matrix and its inverse matrix has received extensive attention and research, and a series of estimation formulas have been obtained. The random matrix is divided into a row random matrix and a column random matrix. A non-negative matrix that satisfies both row and column and is 1 is a double random matrix. For example, the unit matrix is a double random matrix.

For the lower bound of the minimum eigenvalue q(A.A^{-1}) of the Had-amard product of matrix A and its inverse A^{-1}, it is concluded that:

$$q\left(A \circ A^{-1}\right) \geq \frac{1}{n} \tag{1}$$

And guess:

$$q\left(A \circ A^{-1}\right) \geq \frac{2}{n} \tag{2}$$

(3) The form is simple and the calculation is easy, but when the order of the matrix A is large, the effect obtained by the estimation formula is not good. Give improved results:

$$q\left(A \circ A^{-1}\right) \geq \max\left\{1-\rho(J)^2, \frac{1+\rho(J)^{\frac{1}{n+2}}}{1+(n-1)\rho(J)^{\frac{1}{n+2}}}\right\} \tag{3}$$

When the order n is large, the equation (4) improves the equation (3), but since the ρ(J) calculation is more complicated, it is derived from the elements of the matrix:

$$q\left(A \circ A^{-1}\right) \geq \min_{i \in N}\left\{\frac{a_{ii}-s_i R_i}{1+\sum_{j \neq i} s_{ji}}\right\} \tag{4}$$

Among them:

$$\begin{gathered} R_i = \sum_{j \neq i}\left|a_{ij}\right|, d_k = \frac{R_k}{\left|a_{kk}\right|} \\ j \neq i, s_i = \max_{j \neq i}\left\{s_{ij}\right\}, i \in N \end{gathered} \tag{5}$$

The matrix eigenvalue traditional error bound estimate is described as the distance from the eigenvalue of the original matrix to the eigenvalue of the perturbation matrix. The characteristic polynomial coefficients are at the core of quantum physics applications, and in particular it provides such information. If the matrix A is a normal matrix, the value of the binomial becomes smaller. At this point, the elementary symmetry function is for all singular values rather than the maximum part, and better coefficient constraints can be obtained. Using the characteristics of the irreducible matrix, the new lower bound of the minimum eigenvalue of Hadamard product of one matrix and the inverse matrix of another matrix is given [10]. The research method is extended from general matrix to special matrix. For example, the perturbation bounds of normal matrix and Hermite positive definite matrix are derived, which are better than those of general matrix. From the point of view of eigenvalue, the elementary function is characterized and its expansion is given. From lemma, it is known that the perturbation bound of the coefficients of the characteristic polynomial can be converted to the error bound of the function t_k. Then the relative perturbation bound of the coefficients is discussed by using the first-order absolute perturbation bound of the predecessors.

The numerical experiments show that these new conclusions are superior to the existing results for some specific matrices, and the range of the lower bound can be narrowed by combining the existing conclusions. Firstly, the absolute norm perturbation bounds of the characteristic polynomial coefficients are discussed from the determinant expansion, and their first-order absolute perturbation condition numbers are given. The theorem is improved from general matrix to special matrix, from eigenvalues to generalized eigenvalues. From complexity to simplification, step by step, a new disturbance world is drawn. They are more precise or easier to calculate than previous conclusions.

4. Conclusion

Under the condition of operator norm and generalized eigenvalue, the absolute perturbation bound of the characteristic polynomial coefficients is studied. For the upper bound of the maximum eigenvalue of the Hadamard product of the matrix and the inverse matrix, how to use the matrix element to characterize the radius of the non-negative matrix Hadamard product spectrum The upper bound estimate and so on. This paper mainly studies the maximum eigenvalues of two non-negative matric Hadamard products and the upper bound of the spectral radius and the lower bound of the minimum eigenvalue of Hadamard product of one matrix and the inverse matrix of another matrix. For the non-negative matrix dominated by the diagonal of the column, there are some results, using the Gerschgorin disk theorem and the Brauer egg theorem. Elliptical regions, the estimation of the Hadamard product spectrum radius of non-negative matrices have many results. Some results are obtained by using Cauchy-Schwith inequality. The importance of matrix Hadamard product has been recognized by more and more mathematicians. Non-negative matrices and matrices are two special types of matrices. Two new estimators of maximum eigenvalues and upper bounds of spectral radius of Hadamard product of two nonnegative matrices are given by using the method of estimating eigenvalues of matrices in elliptic regions. The example shows that the estimation formula is more accurate than the existing one. Although the conclusion only depends on matrix elements, it has not been proved to be more accurate than the existing conclusions, which also leaves a lot of room for future research.

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